Estimating Discharge Measurement Uncertainty Using the Interpolated Variance Estimator

Timothy A. Cohn[1]; Julie E. Kiang[2]; and Robert R. Mason Jr., P.E., M.ASCE[3]

Abstract: Methods for quantifying the uncertainty in discharge measurements typically identify various sources of uncertainty and then estimate the uncertainty from each of these sources by applying the results of empirical or laboratory studies. If actual measurement conditions are not consistent with those encountered in the empirical or laboratory studies, these methods may give poor estimates of discharge uncertainty. This paper presents an alternative method for estimating discharge measurement uncertainty that uses statistical techniques and on-site observations. This interpolated variance estimator (IVE) estimates uncertainty based on the data collected during the streamflow measurement and therefore reflects the conditions encountered at the site. The IVE has the additional advantage of capturing all sources of random uncertainty in the velocity and depth measurements. It can be applied to velocity-area discharge measurements that use a velocity meter to measure point velocities at multiple vertical sections in a channel cross section. **DOI:** 10.1061/(ASCE)HY.1943-7900.0000695.

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Introduction

Tens of thousands of discharge measurements are made annually by the U.S. Geological Survey (USGS), other government agencies, and private entities to quantify the flow in rivers and streams in the United States. Assessing the uncertainty of the measured flow is important to many applications. This paper presents a new technique for estimating the uncertainty of discharge measurements, the interpolated variance estimator (IVE). The IVE method uses a statistical approach to calculate uncertainty by using the individual depth and velocity measurements made across the stream channel. This avoids reliance on results of previous analyses of uncertainty at other locations, where conditions may have been quite different than those during the discharge measurement of interest. IVE can be applied to discharge measurements made using the traditional velocity-area measurement technique. The velocity-area technique is in widespread use, including by the USGS, although other techniques are also often used. Uncertainty in discharge measurements made using other techniques such as a calibrated flume, acoustic Doppler current profiler (ADCP), dilution, or volumetric methods are not discussed in this paper.

To provide a framework for discussion, it is assumed that channel geometry is well defined and that there exists a three-dimensional system of coordinates on the cross section such that the streamflow velocity, v, is predominantly in the y (longitudinal)-direction, the z-axis points upward, and the x-axis describes horizontal (latitudinal) distance across the channel reach (by convention, usually the distance in from the left edge of the water looking downstream).

The discharge of a channel can be computed as the flow velocity times the flow area. When integrated across the channel

$$Q = \int_A v dA \tag{1}$$

where Q = stream discharge; v = velocity in the downstream direction (perpendicular to the cross section); and A = cross-sectional area.

The velocity-area method of discharge measurement approximates this integral by measuring the width, depth, and velocity at discrete points along the channel cross section (Fig. 1). The total discharge in a channel is estimated by summing the discharge estimated for all the vertical sections (verticals), which together span the width of the channel

$$Q = \sum_{i=1}^{m} A_i v_i = \sum_{i=1}^{m} b_i d_i v_i \tag{2}$$

where Q = measured stream discharge; m = number of verticals across the channel; A_i = cross-sectional area of vertical i; b_i = width of vertical $i \equiv (x_{i+1} - x_{i-1})/2$ with x = horizontal distance of vertical from the edge of water; d_i = average depth of vertical i; and v_i = average downstream velocity in vertical i.

To estimate the average velocity at a vertical, velocity is typically measured at 1, 2, or 3 points in each vertical, depending on the depth and flow conditions. Velocity at these points is used to estimate the average velocity without measuring the full vertical velocity profile. The flow velocity is measured either by mechanical current meter or by acoustic velocity meter.

The USGS uses the midsection method to define vertical subsections using these measurements. The subsection area extends

[1]U.S. Geological Survey, 12201 Sunrise Valley Drive, MS415, Reston, VA 20192.

[2]U.S. Geological Survey, 12201 Sunrise Valley Drive, MS415, Reston, VA 20192 (corresponding author). E-mail: jkiang@usgs.gov

[3]U.S. Geological Survey, 12201 Sunrise Valley Drive, MS415, Reston, VA 20192.

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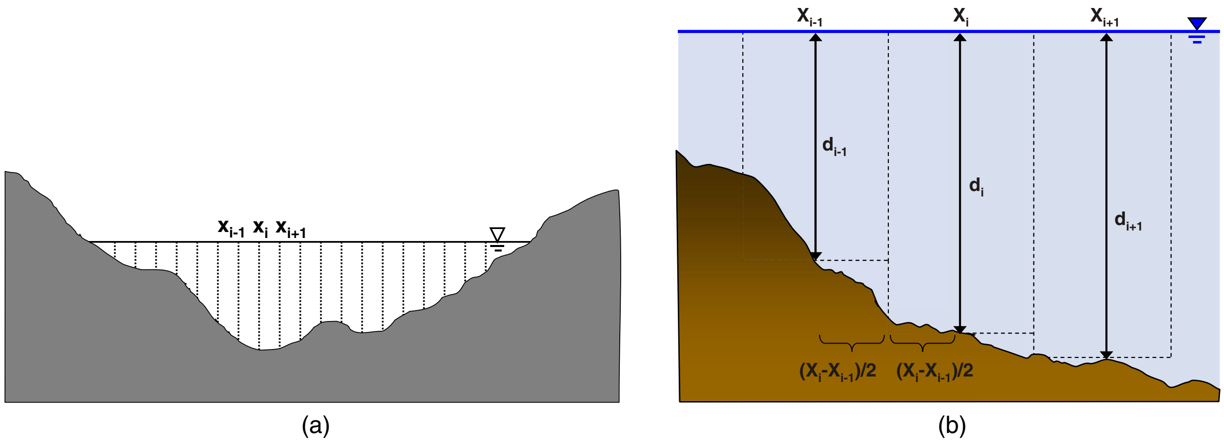

Fig. 1. Illustration of river cross section using the velocity-area method to compute a discharge measurement: (a) vertical sections across river; (b) midsection representation of vertical subsections

half the distance to the preceding and following measurements, as depicted in Fig. 1(b). The measured depth and velocity in the downstream direction are assumed to represent the average depth and average velocity throughout the vertical subsection.

Additional details on discharge measurement techniques used by USGS are described in Rantz et al. (1982) and Turnipseed and Sauer (2010).

Estimating Measurement Uncertainty

Approximations inherent in the velocity-area method include the use of a limited number of verticals and limited sampling (in time and space) of the velocity field within each vertical. These approximations combined with instrument precision and other sources of uncertainty result in uncertainty in the final calculated discharge. For decades, USGS hydrographers have used a qualitative measure of uncertainty by rating each discharge measurement excellent, good, fair, or poor. Although these ratings are subjective, they are based on an evaluation of factors that affect measurement accuracy such as the uniformity of the velocity, the stability of the channel bottom, or other factors that the hydrographer feels may have affected the accuracy of the measurement.

Objective methods of estimating discharge measurement uncertainty have also been developed and are based on identifying and quantifying potential souces of error [Herschy 1971, 1985, 1999; International Standards Organization (ISO) 2007b; Sauer and Meyer 1992]. These include random and systematic errors in measuring b_i, d_i, and v_i at each of the verticals as well as error from approximating the integral in Eq. (1) with a finite number of verticals. Following the notation used by ISO 1088 (2007b), error propagation techniques can be used to derive the relative combined uncertainty of discharge calculated using Eq. (2) as

$$u_Q = \sqrt{u_m^2 + u_s^2 + \frac{\sum_{i=1}^{m}[(b_i d_i v_i)^2(u_{b,i}^2 + u_{d,i}^2 + u_{v,i}^2)]}{\left(\sum_{i=1}^{m} b_i d_i v_i\right)^2}} \qquad (3)$$

where u_Q = relative uncertainty in discharge (fraction); u_b = relative uncertainty in width of a vertical subsection (fraction); u_d = relative uncertainty in average depth of a vertical subsection (fraction); u_v = relative uncertainty in average velocity of a vertical subsection (fraction); u_m = relative uncertainty in discharge measurement owing to approximation of integral using limited verticals (fraction); and u_s = relative uncertainty in discharge measurement owing to calibration errors and other systematic errors (fraction).

ISO 1088 (2007b) and Sauer and Meyer (1992) further disaggregate the velocity uncertainty by identifying individual sources of velocity measurement error. For example, in ISO 1088, the velocity uncertainty at vertical i, $u_{v,i}$, is calculated from three component uncertainties as follows:

$$u_{v,i} = \sqrt{u_{p,i}^2 + \left(\frac{1}{n}\right)(u_{c,i}^2 + u_{e,i}^2)} \qquad (4)$$

where $u_{p,i}$ = relative uncertainty in estimate of mean velocity owing to the limited number of depths at which velocity measurements are made (fraction); n = the number of depths at which velocity measurements are made; $u_{c,i}$ = relative uncertainty in measured point velocities owing to variable responsiveness of current-meter (fraction); and $u_{e,i}$ = relative uncertainty in measured point velocities owing to fluctuations (pulsation) in actual stream velocity (fraction). Sauer and Meyer (1992) add errors owing to oblique flows as an additional component of the velocity uncertainty.

Representative values for each of the identified components of uncertainty are presented in ISO 748 Annex E (2007a) and Sauer and Meyer (1992). These uncertainty estimates are based on empirical and laboratory studies, including work by Herschy (1975), Carter and Anderson (1963), and others. Pelletier (1988) provides an overview of these and many other studies of uncertainty in discharge measurements, including comparisons of the results obtained by various investigators. Although the results were generally consistent, differences in results from various investigators using different equipment at different locations were noted. ISO 1088 (2007b) notes that values for the component standard uncertainties that are based on previous studies should be considered Type B estimates of uncertainty because they do not rely on repeated observations during the actual course of the measurement of discharge. As defined by the Joint Committee for Guides in Metrology (JCGM) (2008), a Type A evaluation of uncertainty is based on statistical analysis of a series of observations. A Type B evaluation of uncertainty uses means other than statistical anlaysis, including information derived from previous measurement data, calibration data, or values taken from reference books.

As with any empirical study, the values of the component uncertainties presented in ISO 748 (2007a) and by Sauer and Meyer (1992) are most likely to be valid under conditions similar to the study conditions. Because these studies do not necesssarily

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encompass all characteristics encountered at individual stream gauging locations, ISO 1088 (2007b, p. 8) recommends that "whenever possible, the user shall determine independently the values of the component uncertainties." Unfortunately, during routine stream gauging, it is not practical to conduct evaluation of component uncertainties using repeat measurements. Although repeated measurements are needed to evaluate alternative methods for estimating the uncertainty of a discharge measurement, this study is motivated by the need for a means of better incorporating on-site measurement data when estimating discharge measurement uncertainty during routine stream gauging.

The remainder of this paper introduces a new method for conducting a Type A analysis of the uncertainty in the depth and velocity components of a discharge measurement based on statistical analysis of the actual quantities measured during the discharge measurement. Instead of identifying individual sources of velocity or depth uncertainty and trying to estimate their magnitude with repeat measurements under controlled conditions, the IVE estimates uncertainty by statistical analysis of the depths and velocities measured at the many vertical sections. The IVE method is tested using Monte Carlo experiments and experiments with measured data, including a set of repeated measurements.

Interpolated Variance Estimator

Approach

In this section, the Interpolated Variance Estimator (IVE) is described as a method for estimating uncertainty owing to random errors in depth and velocity estimates. The estimation of uncertainty in depth is first presented in detail as an example, and then the analagous method for estimating uncertainty in velocity is briefly summarized.

Following Eq. (2), estimates of the average depth and average velocity of each vertical are needed in order to calculate discharge. The velocity-area method assumes that point measurements provide an unbiased estimate of average depth and average velocity for each vertical. u_d and u_v, as calculated by the IVE method, are estimates of the relative uncertainties in these estimates of average depth and average velocity. They do not represent the uncertainty in the measurement of depth or velocity at a single point.

By using a limited number of verticals to represent the entire channel cross section, the velocity-area discharge measurement technique requires that the number of verticals is adequate to capture all of the gross features of the depth and velocity profiles. If the number of verticals is sufficient to approximate the flow integral [Eq. (1)], the depth and velocity measurements will tend to vary smoothly across the channel, and adjacent depths and velocities will vary approximately linearly. Nonlinearities in the measured depth and velocity profiles are an indication that the verticals were not spaced closely enough to properly approximate the flow integral.

Nonlinearities in the measured profiles may also result from measurement error. If, for example, the true depth or velocity profile is exactly linear, measurement error may cause the measured profile to exhibit nonlinearities. Again, nonlinearities in the measured profiles are an indication of measurement error or uncertainty.

The IVE takes advantage of the concept that larger departures from linearity in the measured profiles reflect larger uncertainties. The IVE uses linear interpolation between adjacent verticals to calculate a second estimate of the mean depth (or velocity) for each vertical. The difference between the measured estimate of depth (or velocity) and the interpolated estimate of depth (or velocity) is a measure of the degree to which adjacent verticals depart from linearity. This information is then used to calculate the uncertainty in the estimate of average depth (or velocity) at each vertical. Uncertainty resulting from different sources of random error, including uncertainties arising from the use of a limited number of verticals, are all accounted for by using this approach. This eliminates the need to enumerate and quantify the sources of uncertainty individually. In addition, any interactions between the sources of uncertainty are fully accounted for because they are all lumped into the IVE calculation. Errors that may result from changing flow conditions over the duration of the discharge measurement are not considered.

To employ IVE, random errors in estimating the average depth or velocity are assumed to be independent and the distribution of the errors is assumed to be the same at each vertical. Although it is unlikely that errors are perfectly independent and identically distributed (IID) in real measurements, results from application of the IVE method using real data suggest that they are close enough to being IID that the IVE method is a useful means of estimating discharge measurement uncertainty.

The IVE method applies only to random errors in depth and velocity. Uncertainty owing to random errors in the width measurement (u_b) and uncertainty owing to instrument calibration and other systematic uncertainties (u_s) must be estimated separately using other approaches.

Depth

Errors in estimates of the average depth of a vertical may occur, in part, from positioning the wading rod with varying penetration of the channel bed, from difficulty reading the depth markings on the wading rod, from the effects of drag on suspension cables and weights, or from other circumstances. In addition, errors arise from using a single measurement at the vertical midpoint to represent the mean depth throughout the vertical. These errors may have both a random and a systematic component to them. As noted previously, the IVE estimates the uncertainty owing to random errors, but the uncertainty owing to systematic errors must be estimated separately.

The measured depth, d_i, is assumed to be an unbiased estimate of the mean depth of vertical i. The measured depth is equal to the true average depth $D(x_i)$ plus an error, ϵ_i

$$d_i = D(x_i) + \epsilon_i \tag{5}$$

Likewise, the measured depth of adjacent verticals can be represented as

$$d_{i-1} = D(x_{i-1}) + \epsilon_{i-1} \tag{6}$$

$$d_{i+1} = D(x_{i+1}) + \epsilon_{i+1} \tag{7}$$

It is assumed that the random errors, ϵ, are identically distributed with mean 0 and variance s_d^2. The standard uncertainty, s_d, is simply the square root of the variance. The relative uncertainty, u_d, at any individual vertical can be estimated by dividing the standard uncertainty, s_d, by the measured depth of the vertical.

A second independent and first-order unbiased estimate of the mean depth of vertical i can be obtained by interpolating between the measured depths of the two adjacent verticals as follows:

$$d_{i,\mathrm{est}} = \omega_i d_{i-1} + (1 - \omega_i) d_{i+1} \tag{8}$$

where

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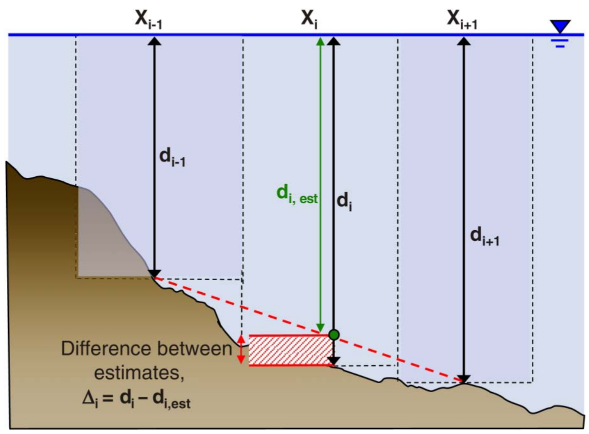

Fig. 2. Measured depth, d_i, provides an estimate of the average depth of the vertical; a second estimate of the depth, $d_{i,\text{est}}$, can be obtained at each vertical by interpolating from adjacent verticals; Δ is the difference between this original measurement, d_i, and the interpolated estimate, $d_{i,\text{est}}$

$$\omega_i = \frac{x_{i+1} - x_i}{x_{i+1} - x_{i-1}} \tag{9}$$

and x_i = distance across the stream channel from an arbitrary starting point, usually at the edge of the water (Fig. 2).

The difference between d_i and $d_{i,\text{est}}$ can be calculated at each vertical and is defined as follows:

$$\Delta_{i,d} = d_i - d_{i,\text{est}} \tag{10}$$

This equation can also be expanded to

$$\Delta_{i,d} = D(x_i) + \epsilon_i - \omega_i[D(x_{i-1}) + \epsilon_{i-1}] - (1 - \omega_i)[D(x_{i+1}) + \epsilon_{i+1}] \tag{11}$$

Note that Δ_i will tend to be large if the measurement errors are large.

If the expected value of both d_i and $d_{i,\text{est}}$ are equal to the true mean depth, then the expected value of the difference between them, $\Delta_{i,d}$, is zero as shown by the following:

$$E[\Delta_{i,d}] = E[d_i] - E[d_{i,\text{est}}]$$

$$E[\Delta_{i,d}] = D(x_i) - D(x_i)$$

$$E[\Delta_{i,d}] = 0$$

The variance of $\Delta_{i,d}$ is

$$\begin{aligned}\text{Var}[\Delta_{i,d}] &= \text{Var}\{D(x_i) + \epsilon_i - \omega_i[D(x_{i-1}) + \epsilon_{i-1}] - (1 - \omega_i)[D(x_{i+1}) + \epsilon_{i+1}]\} \\ &= \text{Var}[\epsilon_i - \omega_i\epsilon_{i-1} - (1 - \omega_i)\epsilon_{i+1}] \\ &= \text{Var}[\epsilon_i] + \omega_i^2\text{Var}[\epsilon_{i-1}] + (1 - \omega_i)^2\text{Var}[\epsilon_{i+1}] \\ &= s_d^2 + \omega_i^2 s_d^2 + (1 - \omega_i)^2 s_d^2 = 2(1 - \omega_i + \omega_i^2)s_d^2\end{aligned}$$

The variance of $\Delta_{i,d}$ can also be estimated as the sample variance

$$\text{Var}[\Delta_{i,d}] = \frac{1}{(n-4) - 1}\sum_{i=3}^{n-2}(\Delta_{i,d} - E[\Delta_{i,d}])^2 \tag{12}$$

The sample variance is calculated for $\Delta_{i,d}$ by using verticals 3 to $n - 2$ because the variance at the channel edges may not conform to the variance elsewhere in the channel. This leaves $n - 4$ values of $\Delta_{i,d}$ in the sample. Because $E[\Delta_{i,d}]$ is equal to zero, this equation reduces to

$$\text{Var}[\Delta_{i,d}] = \frac{1}{(n-4) - 1}\sum_{i=3}^{n-2}(\Delta_{i,d})^2 \tag{13}$$

By combining the two expressions for the variance of $\Delta_{i,d}$, the standard uncertainty in the average depth of a vertical subsection owing to random errors can be calculated as the following:

$$s_d = \sqrt{\left(\frac{1}{n-5}\right)\sum_{i=3}^{n-2}\frac{(\Delta_{i,d})^2}{2(1 - \omega_i + \omega_i^2)}} \tag{14}$$

The instrument resolution can be used as a lower limit to s_d.

Velocity

As with depth, velocity measurements are made for each vertical, and the estimate of average velocity in a vertical subsection has many possible components of error. These include instrument error, the length of the instrument's exposure to the flow (error arising from fluctuations in the velocity), and errors in extrapolating mean velocity of the entire vertical subsection from 1–3 point measurements in each vertical. Each of these sources of error includes a random component, which tends to decrease the precision with which velocity is measured at any vertical. To the degree that these variations cause departures from a smooth velocity profile across the channel, they are detected by IVE as increases in uncertainty.

For most measurements, IVE adequately captures the effects of turbulence in increasing the uncertainty in velocity. If the period used to sample velocity is of insufficient duration, a stable velocity will not be available for each vertical, resulting in more variation of velocity at adjacent verticals and a higher estimated uncertainty. The length of time that transpires between measurements of velocity at adjacent verticals is assumed to be sufficient such that each measurement can be considered independent of the other, despite turbulent structure in the flow. If the time scale of the turbulence is on the order of the length of the entire measurement, including all verticals, then there may be bias in the measured velocities that the IVE method cannot detect.

The uncertainty calculated using the IVE method does not account for systematic errors that may arise by assuming an inappropriate vertical velocity profile to extrapolate mean velocity from limited measurements in each vertical profile. These, and other systematic errors resulting from instrumentation or hydrographer technique, are estimated separately when calculating the combined uncertainty.

Analogous to the method employed in the previous section for depth, the downstream velocities are estimated in two independent ways. At each vertical there is (1) the measured velocity for the vertical and (2) the velocity interpolated from adjacent verticals. Following the same derivation used for depth errors, the standard uncertainty of the estimate of average velocity in each vertical, s_v, is

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$$s_v = \sqrt{\left(\frac{1}{n-5}\right)\sum_{i=3}^{n-2}\frac{(\Delta_{i,v})^2}{2(1-\omega_i+\omega_i^2)}} \quad (15)$$

Again, the instrument resolution can be used as the lower limit to s_v and the relative uncertainty, u_v, can be estimated by dividing s_v by the measured velocity of the vertical.

Width

An interpolated estimate of the width of each subsection cannot be obtained. Consequently, the IVE cannot be used to estimate the random error in measurements of width. Instead, uncertainty in width is estimated using previously published estimates. Sauer and Meyer (1992) suggest that the width errors are generally considered to be less than 1%. ISO 1088 (2007b), Annex G suggests that the standard uncertainty in the measurement of width should be no greater than 0.5%.

Uncertainty Owing to Calibration Uncertainty and Other Systematic Errors

Uncertainty is introduced to the discharge measurement by instrument calibration because even a well-calibrated instrument is subject to small systematic errors. A miscalibrated instrument will introduce a significant nonrandom, or systematic, error to the measurements because each measurement will be influenced by the same miscalibration. Instrumentation bias could result from faulty or inappropriate meter ratings; damaged, bent or misaligned meter cups or transducers; or other factors. When the amount of this error is unknown, an additional source of uncertainty results. Similarly, differences in how individuals operate equipment or read measurements may result in systematic uncertainties. Bias in hydrographer technique might include habitual misorientation or movement of the wading rod or omission or misidentification of angular oblique flows. These systematic errors are in addition to the random errors that are estimated using the IVE method.

There may be systematic uncertainties in each of the components of the discharge measurement, and they can be combined into one quantity

$$u_s = \sqrt{u_{sb}^2 + u_{sd}^2 + u_{sv}^2} \quad (16)$$

where u_s = total systematic uncertainty; u_{sb} = systematic uncertainty in the width measurement; u_{sd} = systematic uncertainty in the depth measurement; and u_{sv} = systematic uncertainty in the velocity measurement.

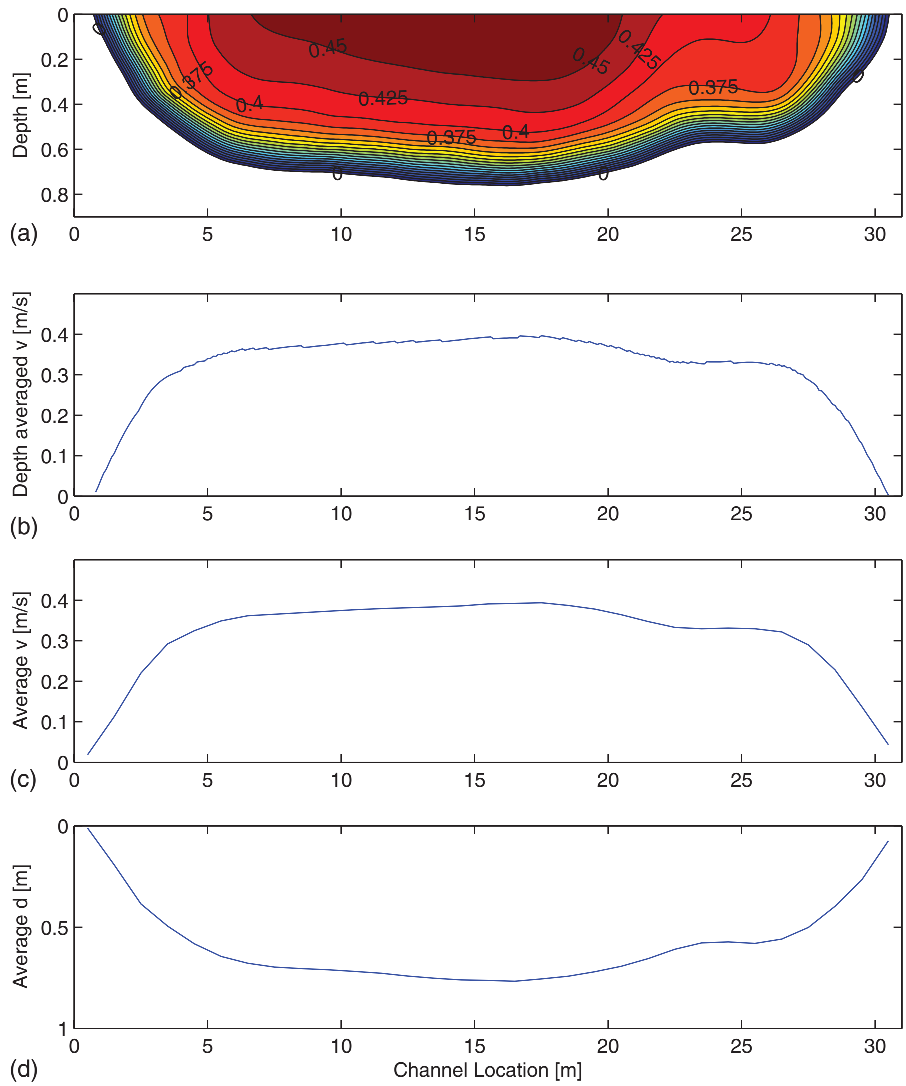

Fig. 3. (a) Channel geometry and velocity field (velocity contours are in m/s); (b) continuous depth-averaged velocity calculated from the full velocity field; (c) average depth for each of 31 verticals; (d) average velocity for each of 31 verticals

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Uncertainty owing to calibration uncertainty or other systematic uncertainty are also described within ISO 748 (2007a), ISO 1088 (2007b), and Sauer and Meyer (1992). When using the IVE method, values for u_s can be estimated following recommendations in ISO 748 and ISO 1088. The ISO standards suggest that u_s can be assigned a value of 1% as an "estimated practical value." Sauer and Meyer suggest a value of 0.5% each for width, depth, and velocity, yielding a total of 0.87% when combined using Eq. (16).

Combined Uncertainty

The combined uncertainty is calculated using the same equation that is used by ISO 1088 (2007b), except that u_m is omitted, because it is incorporated into the IVE estimates of u_d and u_v

$$u_Q = \sqrt{u_s^2 + \frac{\sum_{i=1}^{m}[(b_i d_i v_i)^2(u_{b,i}^2 + u_{d,i}^2 + u_{v,i}^2)]}{(\sum_{i=1}^{m} b_i d_i v_i)^2}} \quad (17)$$

Monte Carlo Experiments: Synthetic Measurements

In this experiment, known errors are introduced to synthetic discharge measurements in a hypothetical channel. In this Monte Carlo experiment, the IVE is used to estimate the uncertainty of 10,000 synthetically generated measurements. Because the imposed errors are known, the distribution of the known errors can be compared with the IVE-estimated uncertainty.

Fig. 3(a) depicts the channel geometry and full velocity field of the hypothetical channel used in the experiment. The channel is approximately 30 m in width and 0.8 m deep at the thalweg. The depth averaged velocity is shown in Fig. 3(b). The channel was split into 31 verticals to simulate a velocity-area discharge measurement. The resulting average depth and velocity for each vertical are shown in Figs. 3(c and d).

The measured depth and velocity at each vertical are generated from a gamma distribution with specified mean and variance. A normal distribution was initially considered because the lumped errors are the sum of many different quantities. According to the central limit theorem, the errors are then approximately normally distributed. However, because neither depth nor velocity can be less than zero, a gamma distribution is used to prevent any instances of negative depth or velocity when the specified variance is large, while also retaining a bell-shaped distribution.

Gamma-distributed random errors with standard deviation ranging from 0.01 to 0.2 m for depth and from 0.01 to 0.15 m/s for velocity are added to each vertical measurement. For each level of imposed error, the IVE method is used to estimate uncertainty in 10,000 realizations of a simulated discharge measurement.

No width measurement errors or systematic errors were introduced to the synthetic measurements. Consequently, the systematic uncertainty and width uncertainty are both set to zero in the calculation of the combined uncertainty of the synthetic measurements. Also, no lower limit to the depth or velocity uncertainties was used in any of the synthetic experiments.

The true discharge measurement error for each realization is defined as the difference between the discharge in the (perturbed) synthetic measurement and the true discharge as calculated using the full channel geometry and velocity field. The true uncertainty is then calculated as the standard deviation of these measurement errors over the set of 10,000 synthetic measurements. For each individual synthetic measurement, the IVE is used to estimate u_d and u_v, and the combined discharge measurement uncertainty is calculated using Eq. (17).

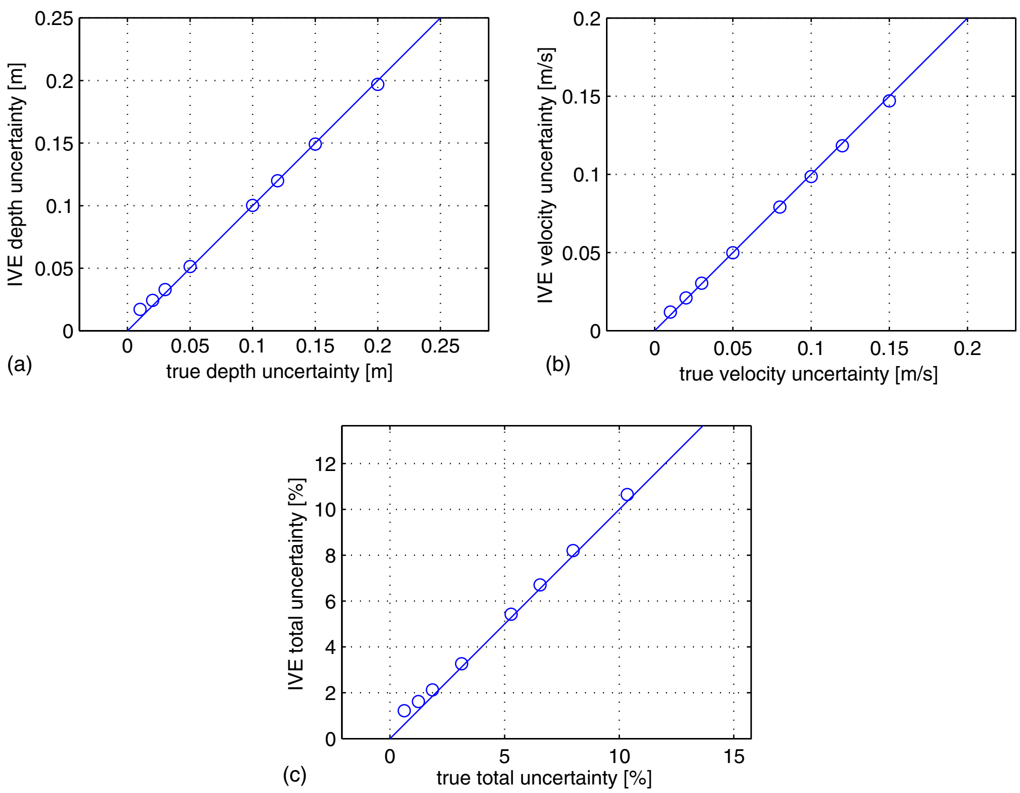

Fig. 4. IVE estimates of (a) depth and (b) velocity uncertainty are compared with the errors actually imposed upon each measurement; (c) IVE estimate of combined (total) measurement uncertainty is compared to the true uncertainty, calculated as the standard deviation of known errors

Figs. 4(a and b) show the mean of the IVE's estimates of depth and velocity provides a good estimate of the errors imposed on the depth and velocity. Fig. 4(c) shows that the mean of estimates of the combined uncertainty are very close to the true uncertainty. There appears to be a small bias, with the total uncertainty slightly overestimated by IVE. This bias can be attributed in part to small inaccuracies in estimates of depth and velocity uncertainty. Particularly for larger combined uncertainty, part of the bias also arises because the true depth and velocity of each vertical subsection are not known when calculating the uncertainty, and this part of the bias would remain even if IVE's estimates of s_d and s_v were perfect.

Natural Channel Experiments

Results for Fox River, Illinois, Using ADCP Measurement

In this set of experiments, ADCP measurements were utilized to define true depth and velocity profiles for the Fox River near Montgomery, Illinois (USGS station number 05551540). IVE was then used to estimate the uncertainty of measurements synthesized by subsampling depth and velocity from the ADCP-defined profiles.

A 600 kHz Rio Grande ADCP using Water Mode 11 and Bottom Mode 5 was used to collect between 600 and 800 ensembles on each of four passes across the river. Each ensemble consists of depth and velocity measurements at a specified cross-sectional location, x, in the channel. The depth at each location was estimated as the average of the depth measurements collected from the ADCP's four beams. The vertically averaged velocity at each location was calculated by fitting the array of point velocity measurements at the vertical location to a power function. ADCP instrumentation is unable to collect data close to the shore because the channel bottom and sides interfere with the acoustic measurement. For this experiment, the channel area was assumed to consist only of the measured portion of the channel away from the channel edges. This measured flow averaged approximately 16.4 m^3/s (580 ft^3/s).

Although ADCP measurements and the individual ensembles collected during ADCP measurements are subject to uncertainty,

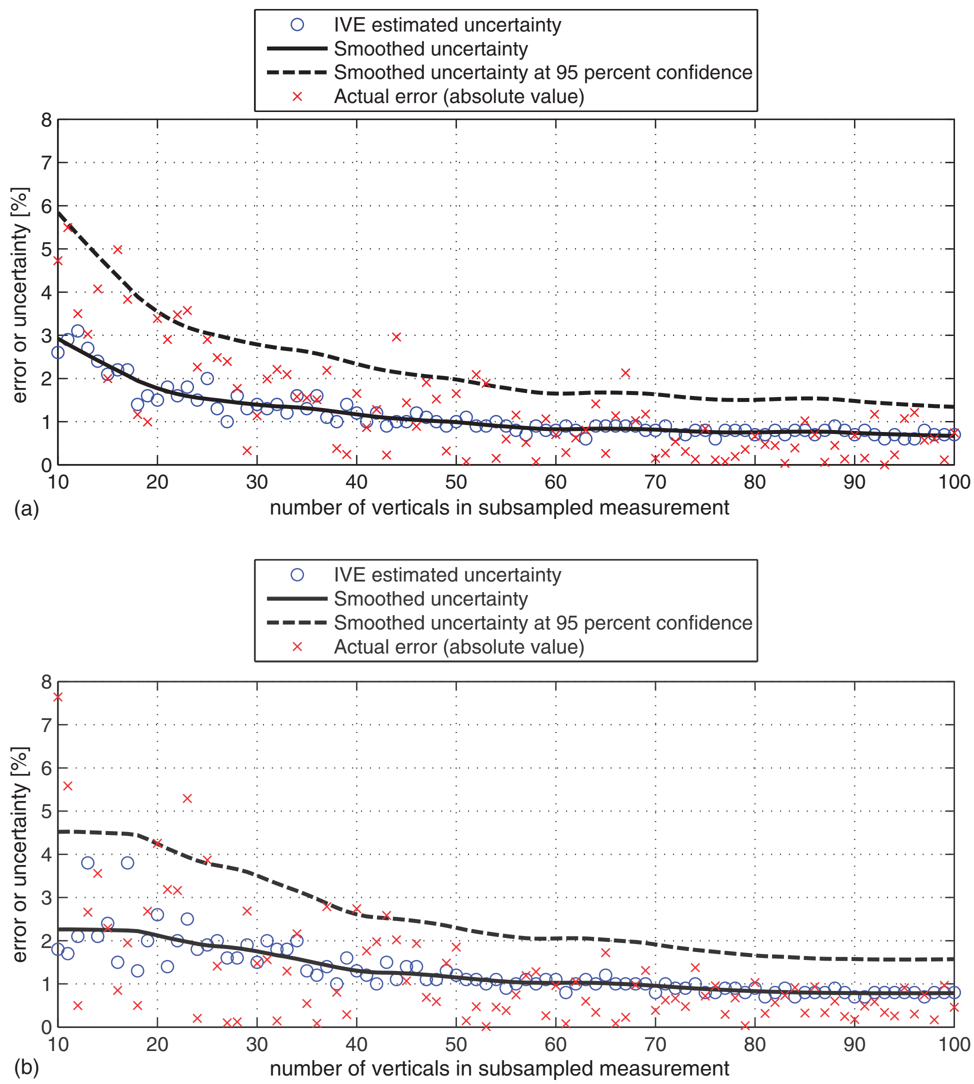

Fig. 5. ADCP data were used to define the true depth and velocity for the Fox River at Montgomery, Illinois; using between 10 and 100 evenly spaced verticals subsampled from the true profiles; the true error and the IVE-estimated uncertainty are compared; (a) and (b) show results from two different ADCP transects

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the ADCP measurement data are used in this study to define a plausible channel cross section with a large amount of detail. The true discharge is defined as that calculated by using the depth and velocity at all of the available vertical ensembles. Then, discharge measurements were simulated by subsampling between 10 and 100 evenly spaced verticals interpolated from the original data. The uncertainty of each of these simulated discharge measurements was calculated by using IVE. Because the only errors are attributable to subsampling of the depth and velocity measurements, the width uncertainty and the systematic uncertainty were both set to zero. The true error of each simulated measurement was calculated with respect to the true discharge.

Fig. 5 shows examples of results of this experiment. The open circles indicate the IVE-estimated uncertainty, and the "x" symbols indicate the true error of each simulated measurement. The solid black line shows a smoothed curve through the IVE-estimated uncertainty for different measurements. The dashed black line approximates the 95% confidence interval for the IVE-estimated uncertainty. Approximately 95% of the actual measurement errors should be less than this confidence interval, with just 5% of the errors larger. Overall, for the four ADCP transects, approximately 6% of the actual errors lie outside the dashed black line.

Results for Watauga River, North Carolina

Data from discharge measurements on the Watauga River at Sugar Grove, North Carolina (USGS station number 03479000), were used to test the IVE estimates of uncertainty for typical discharge measurements. The Watauga River drains 238.5 km^2 (92.1 mi^2) at the stream gauge and is not regulated by upstream reservoirs. As part of a training exercise, a mix of novice and experienced stream gaugers made 21 wading discharge measurements on October 22, 1991. Over the course of the day, the discharge calculated by applying the station's rating curve to measured stage varied between 0.85 and 0.88 m^3/s [30–31 ft^3/s, a 3.3% difference (Fig. 6)].

Because flow was steady, nearly the same flow would have been measured each time had each measurement been without error. The standard deviation of the measurements quantifies the variability observed among the measurements. Assuming that the uncertainty of each measurement was the same (because similar cross sections, equipment, and technique were used for each measurement), the standard deviation is a measure of the true uncertainty of any individual measurement. This data set thus allows an assessment of IVE uncertainty estimates for measurements that follow standard USGS protocols.

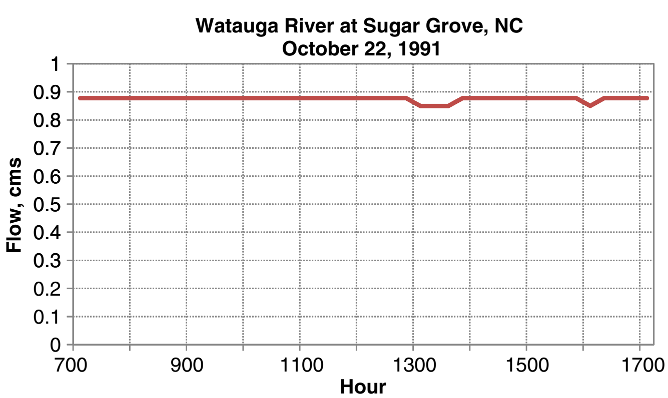

Fig. 6. Flow on the Watauga River at Sugar Grove, North Carolina, on October 22, 1991

One of the discharge measurements used only 12 verticals, well below the 25–30 verticals typically recommended for USGS measurements (Rantz et al. 1982; Turnipseed and Sauer 2010). This measurement was eliminated from further consideration to maintain a more consistent set of measurements. The remaining 20 measurements each used between 24 and 38 verticals. Price Pygmy meters were used for all but one measurement, for which a Price AA meter was used.

The true discharge for the period during which measurements were made is calculated as the mean of the 20 discharge values [0.93 m^3/s (32.7 ft^3/s), slightly higher than the flow calculated from the rating curve]. The true uncertainty of each measurement is assumed to be the same and is estimated as the standard deviation of the 20 measured discharge values, or 0.05 m^3/s (1.8 ft^3/s, 5.5%).

The IVE method was used to estimate the uncertainty of each discharge measurement. The width uncertainty was assumed to be 0.5%, and the systematic uncertainty was assumed to be 1%. The lower limit to the depth uncertainty was set at 0.003 m (0.01 ft) based on the resolution of the depth markings on the wading rod used in the measurements. The lower limit to the velocity uncertainty was set at 0.009 m/s (0.03 ft/s), based on the rating table used for Pygmy meters and a measurement time of 40 s. The IVE-estimated depth and velocity uncertainties were consistently higher than these lower bounds.

The average uncertainty of the measurements calculated using IVE is 4.9%. The estimates for individual measurements range from 4.0 to 6.0%.

Summary and Conclusions

The IVE was introduced as a new method for estimating the uncertainty of discharge measurements made using the velocity-area method. Uncertainty estimates are based on a statistical analysis of on-site measurements of depth and velocity. This new IVE method is possible because a velocity-area discharge measurement utilizes many measurements of velocity and depth across the channel. Rather than using results from a series of controlled experiments conducted elsewhere, IVE is able to use data collected during the measurement to estimate uncertainty given the on-site conditions.

The IVE method was successful in approximating the introduced errors in synthetic measurements used in the Monte Carlo experiment. It also provided reasonable estimates of uncertainty when detailed channel information from an ADCP was used to provide estimates of the true uncertainty. IVE's estimates of uncertainty also compared well to the standard deviation of a set of real discharge measurements on the Watauga River in North Carolina.

Advantages

- The IVE-estimated uncertainty is calculated based on the actual measurements that were made at that site and not on empirical estimates that may or may not fit the site conditions.
- All random sources of uncertainty in each component of the discharge equation are captured using the IVE approach. There is no need to quantify individual sources of uncertainty.
- A computer program can easily calculate the uncertainty of a discharge measurement from actual depth and velocity measurements taken at each vertical. The estimate of the uncertainty of a discharge measurement calculated in this fashion is not subject to the hydrographer's subjective assessment of the conditions and resulting uncertainty.

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Limitations

- IVE's ability to estimate uncertainty depends upon the assumption that depth and velocity errors are independently and identically distributed across the channel. If actual conditions are far from this ideal, IVE may not provide good estimates of uncertainty.
- The IVE approach does not enhance undertanding of the sources of uncertainty (aside from that attributable to depth versus velocity) and thus does not provide insight into strategies that might be employed to reduce uncertainty in discharge measurements.
- As with other methods, estimates of width uncertainties and systematic uncertainties must be obtained from empirical or laboratory results.

Although the IVE method shows promise as a new means of estimating the uncertainty of velocity area discharge measurements, additional testing is needed to further validate the method. The focus of additional testing will be to evaluate IVE's performance under real-world conditions. Additional sets of discharge measurements like the one described in this study for the Watauga River, North Carolina, will be used in future testing. IVE estimates will also be compared to uncertainty estimates using other techniques, such as the ISO method.

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